Internet Appendix for "Is the Credit Spread Puzzle a Myth?"

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Abstract

This appendix reports additional analysis and robustness checks for Bai, Goldstein, and Yang (2019), "Is the Credit Spread Puzzle a Myth?"

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A1. The Black-Cox model using market-to-book adjustment of 0.83 for C-rated debt

We estimate the median value of the market-to-book ratio across all C-rated debt in our corporate bond data to be 0.83. Hence, we perform a robustness check by setting the market-to-book adjustment to 0.83, rather than 0.72, for C-rated debt. Table A1 reports the results. The results are extensions of Tables 3 and 6 in the main text. Panel A reports the cumulative default rates across credit ratings and horizons. Panel B reports the average CDS spreads. The results are similar to the ones when we set the market-to-book adjustment to 0.72 in the main text.

A2. Default boundary estimation based on default rates from 1940 to 2017

We construct the cumulative default rates across credit ratings and horizons (1−20 years) using the defaults from 1940 to 2017 recorded in the Moody's default database. We estimate the default boundary based on these default rates using the FS approach. Table A2 reports the results. Then, we compute the model-predicted CDS spreads and compare them with the data (2002−2017) in Table A3.

A3. Cross-sectional and time-series variations of the default boundary in the jump-diffusion structural model

We investigate cross-sectional and time-series variations in model-implied default boundaries for our jump-diffusion structural model using the following approach. First, we specify the default boundary (as a ratio of the face value of debt) as a logistic function of certain variables ($X_{i,t}$) that can vary across firms-i or time-t (i.e., we assume the following functional form for the scaled default boundary $d_{i,t} = \frac{V_{B,i,t}}{F_{i,t}}$):

$$d_{i,t} = \frac{1}{1 + e^{-A-BX_{i,t}}}. \tag{1}$$

The reason we choose a logistic function is to impose the economically reasonable restriction that the default boundary is constrained to the region $d_{i,t} \in (0, 1)$. We then identify the parameters (A, B) by minimizing the RMSE, averaged across all CDS spreads in our sample. Because computing sample RMSE as a function of $d_{i,t}$ takes a considerable amount of time, we compute this function on a lattice and then use interpolation. We estimate the standard errors of the coefficients (A, and B) using the block bootstrap method to adjust for correlations across firms and time.

Table A4 reports the results. We consider several choices for the variable $X_{i,t}$ to investigate whether or not there is cross-sectional and/or time-series variation in the location of the default boundary. In Panel A, we report the results for two firm-level variables: i) distance-to-default (DD) and ii) credit ratings. DD is defined as

$$DD = \frac{\log \frac{A}{D} + \left(r_f - \delta - \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}}, \tag{2}$$

where A is the sum of market equity and market debt, D is the book debt, r_f is the one-year risk-free rate, δ is the payout ratio, and σ is the asset volatility. In addition, we set $T = 1$ year. When estimating the model with credit rating as the dependent variable, we assign $X_{i,t}$ a number for every credit rating as follows AAA (1), AA (2), A (3), BBB (4), BB (5), B (6), and C (7).

In Panel B, we report the results for five time-series variables: i) average BBB-AAA credit spread, ii) the cyclically adjusted price-to-earnings ratio (CAPE), iii) the one-year risk-free rate, iv) aggregate leverage ratio computed as the cross-sectional mean of leverage ratio measured by the ratio of book debt and the sum of book debt and market equity, and v) the cross-sectional mean of asset volatility (σ_i) implied by the jump-diffusion structural model. All of these variables are estimated at the monthly frequency.

Among all of these firm-level and business-cycle variables, we do not find a single parameter estimate for B that is statistically significant. We therefore conclude that the default boundary (over face debt) $d_{i,t}$ does not appear to vary significantly across ratings/DD or across time.

Fig. A1 and A2 provide visual support for the constant default boundary specification. In panel A (B) of Fig. A1, the point estimate for the default boundary is expressed as a function of DD (credit rating), along with plus or minus one standard deviation. Somewhat similarly, in Fig. A2, for the time-series variables mentioned above, we plot the point estimate for the default boundary as a function of time, along with plus or minus one standard deviation. In all cases, the specification $d = 0.65$ falls within one standard deviation, suggesting that our assumption of a constant default boundary across firms and time is not rejected by the data.

A4. A joint estimation of the jump-diffusion structural model under both the risk-neutral and real probability measures

In the benchmark calibration, we identify the optimal (d, λ^Q) using the entire sample by minimizing the RMSE. We obtain a very sharp and well-identified RMSE minimum at $(d = 0.65 \pm 0.01,\ \lambda^Q = 0.20 \pm 0.02)$, as we demonstrate in Table A5, Panel A.

We also estimate the parameters under the risk-neutral (Q) and the real (P) probability measures in a single step by using the following joint error function:

$$\left(d^*, \lambda^{Q*}, \{\sigma_i^*, \eta_{d,i}^{Q*}\}, \theta^*, \beta_d^*\right) = \underset{\left(d,\lambda^Q,\{\sigma_i,\eta_{d,i}^Q\},\theta,\beta_d\right)}{\arg\min} \left\{ \frac{1}{8N} \sum_{i=1}^{N} \sum_{j=1}^{8} \left(\frac{CDS_{i,j,model}}{CDS_{i,j,data}} - 1 \right)^2 + \sum_{a=AAA}^{C} \omega_a \sum_{T=1}^{20} \frac{1}{T} \left(\pi_{a,T}^P(\theta, \beta_d) - \hat{\pi}_{a,T}^P \right)^2 \right\} \quad (3)$$

The first term is the average RMSE of fitting the CDS spreads, and the second term is the weighted-average RMSE of fitting the historical default rates. For a given set of $(d,\ \lambda^Q)$, we choose $(\sigma_i^*,\ \eta_{d,i}^{Q*})$ to minimize the RMSE of the term structure of the CDS spreads for each firm month observation. We then optimize (θ, β_d) to minimize the RMSE of default rates. The total error as a function of $(d,\ \lambda^Q)$ in Eq. 3 is reported in Panel B in Table A5. Compared to our benchmark calibration (i.e., the two-step procedure), we obtain the same estimates for the Q-measure parameters $(d = 0.65,\ \lambda^Q = 0.2)$ but slightly different estimates for the P-measure parameters $(\theta = 0.09, \beta_d = 2.70)$ (compared to $(\theta = 0.11, \beta_d = 2.76)$ in the benchmark case). As the differences in these estimates for θ and β_d are small, it is not surprising that we find model-predicted default rates to be almost the same as the benchmark case reported in Table 10 in the main text.

A5. Statistical test of the models using the data-minus-model CDS spread

To improve the statistical power of our test of the models, we compute the differences (data-minus-model) between i) the CDS spreads in the data and in the jump-diffusion model and ii) the CDS spreads in the data and in the diffusion-based model (see Eq. 34 in the main text).[1] We also compute the bootstrap standard errors of these differences. Table A6 reports the results. In general, we find that the differences are small and statistically insignificant for the jump-diffusion structural model; but large and statistically significant for the diffusion-based model. For example, for the jump-diffusion model, the data-minus-model for mean and median BBB CDS spreads are close to zero with t-statistic less than two. In contrast, for diffusion-based model, the data-minus model for mean (median) BBB CDS spreads is 35 bps (32 bps) with a t-statistic of approximately five.

[1]We thank the referee for this suggestion.

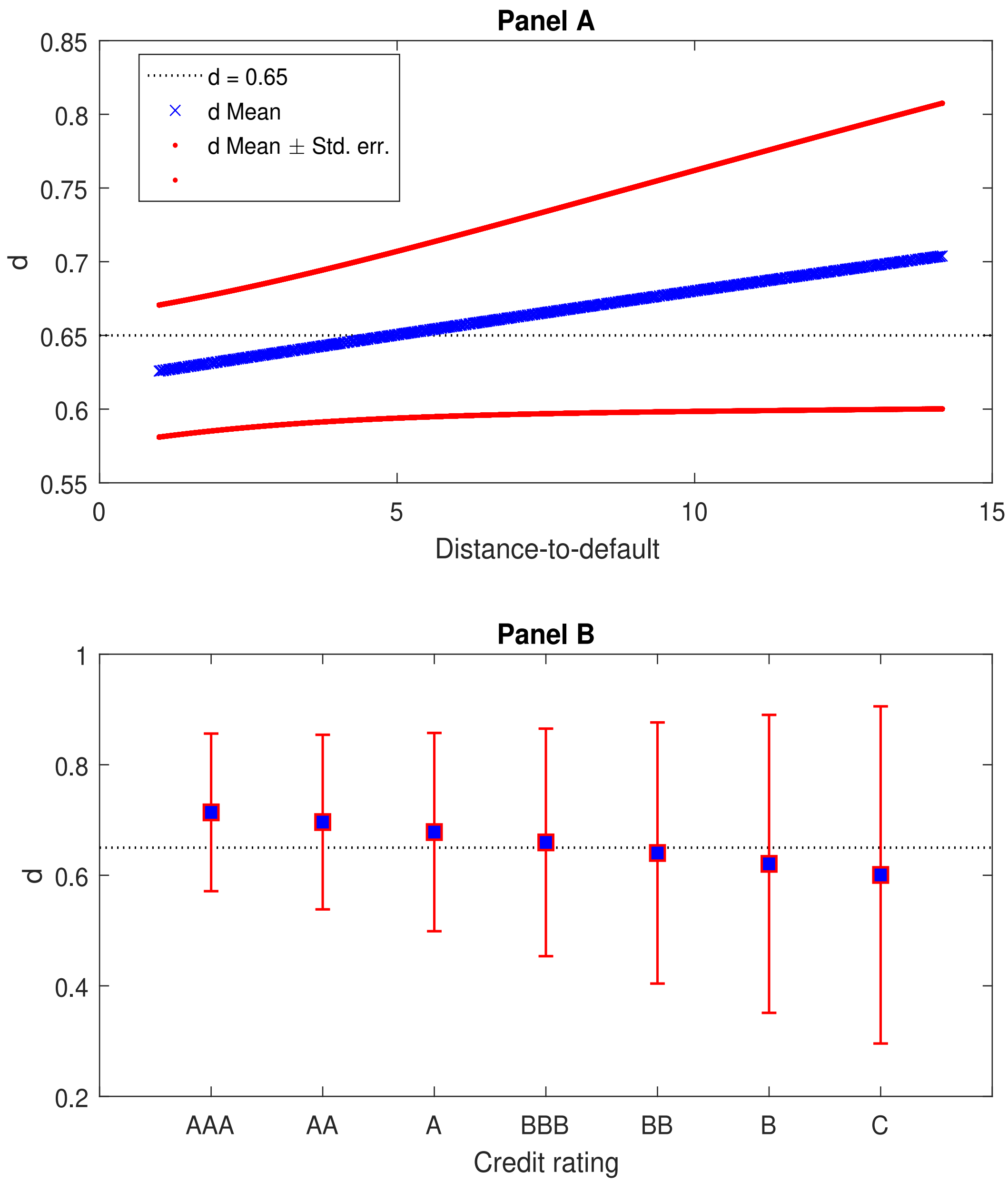

Fig. A1. The default boundary (d) as a function of a cross-sectional variable implied by the jump-diffusion structural model. The default boundary (d) for each firm month observation is computed as a logistic function of its distance to default or credit rating. The parameters of the logistic function are reported in Panel A of Table A4. In Panel A, a firm's default boundary varies across firm-month observations according to its distance-to-default. In Panel B, a firm's default boundary is determined by its credit rating.

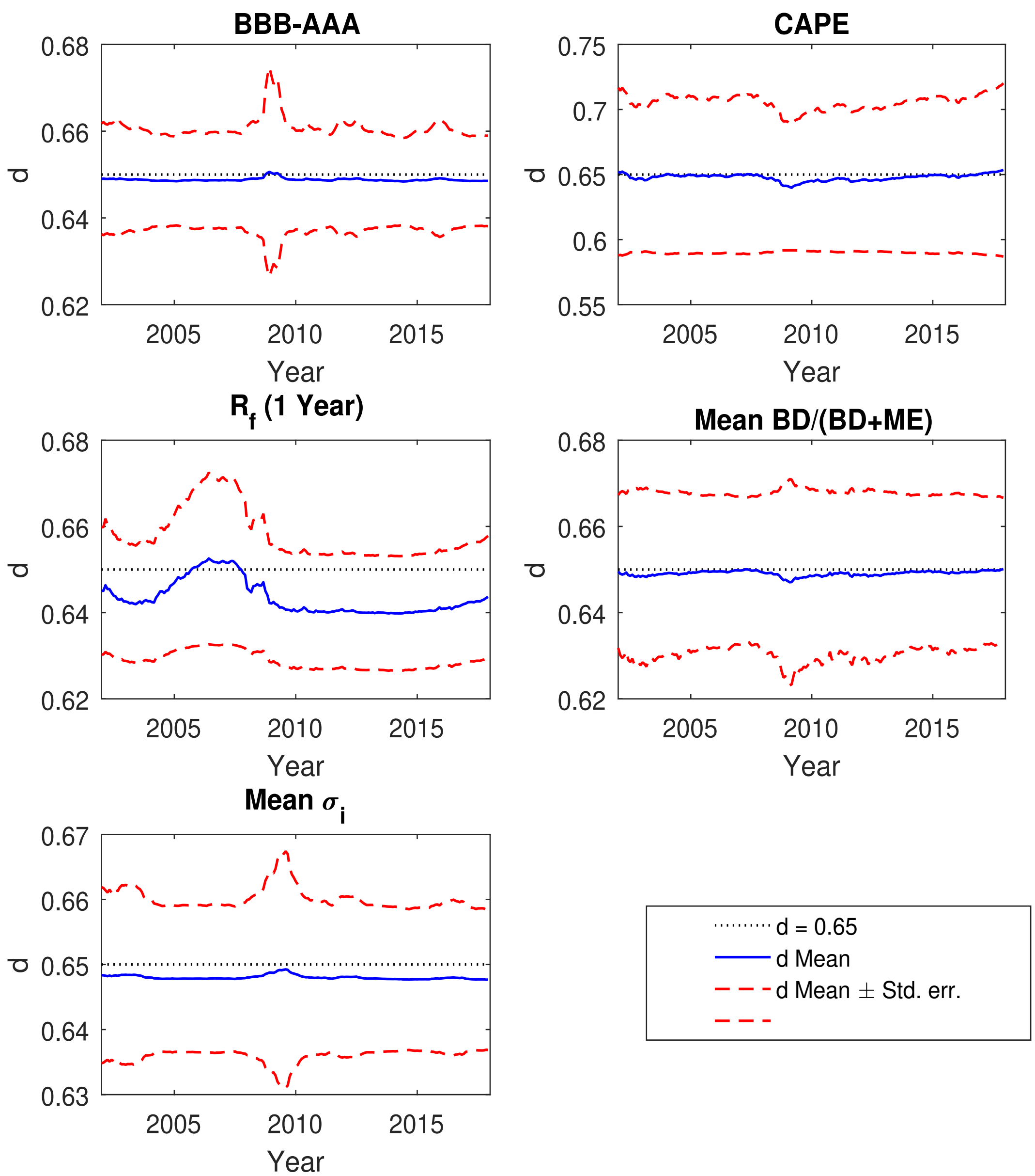

Fig. A2. Time series of the default boundary (d) implied by the jump-diffusion structural model. The default boundary (d) for each firm-month observation is computed as a logistic function of the following contemporaneous macro variables: i) average BBB-AAA credit spread, ii) the cyclically adjusted price-to-earnings ratio (CAPE), iii) the one-year risk-free rate, iv) aggregate leverage ratio computed as the cross sectional mean of leverage ratio measured by the ratio of book debt and the sum of book debt and market equity, and v) the cross-sectional mean of asset volatility (σ_i) implied by the jump-diffusion structural model. The parameters of the logistic function are reported in Panel B in Table A4.

Table A1
Cumulative default rates and CDS spreads in the Black-Cox model using market-to-book adjustment of 0.83 for C-rated debt. This table reports the the Black-Cox model results when we set the market-to-book ratio to be 83% for C-rated debt. We get a default boundary $d = 0.71$ with a standard error equal to 0.02. Panel A reports the cumulative default rates across credit ratings and horizons. Panel B reports the average CDS spreads. Standard errors (s.e.) are computed using the bootstrap method.

Panel A: Cumulative default rates (%)

	Horizon (years)										
	1	2	3	4	5	6	8	10	12	15	20
AAA	0.00	0.05	0.16	0.28	0.42	0.54	0.76	0.95	1.10	1.31	1.64
s.e.	0.00	0.02	0.06	0.10	0.14	0.18	0.26	0.32	0.38	0.44	0.53
AA	0.01	0.05	0.12	0.21	0.32	0.44	0.72	1.01	1.30	1.75	2.54
s.e.	0.01	0.04	0.07	0.11	0.14	0.18	0.26	0.34	0.42	0.53	0.70
A	0.01	0.10	0.26	0.49	0.76	1.04	1.66	2.30	2.96	3.97	5.77
s.e.	0.01	0.06	0.14	0.23	0.32	0.42	0.58	0.73	0.85	1.01	1.22
BBB	0.07	0.43	0.96	1.59	2.27	2.97	4.36	5.73	7.04	8.91	11.88
s.e.	0.04	0.18	0.35	0.51	0.65	0.78	0.99	1.16	1.29	1.45	1.67
BB	0.67	2.21	3.91	5.59	7.16	8.62	11.23	13.47	15.41	17.94	21.54
s.e.	0.19	0.55	0.85	1.09	1.28	1.44	1.68	1.87	2.02	2.18	2.39
B	2.27	6.08	9.88	13.33	16.36	18.99	23.34	26.80	29.61	33.03	37.53
s.e.	0.54	1.05	1.32	1.49	1.60	1.68	1.76	1.79	1.79	1.78	1.80
C	7.68	18.85	28.56	35.66	40.65	44.30	49.44	52.97	55.56	58.47	62.01
s.e.	0.94	1.65	1.78	1.64	1.56	1.57	1.64	1.72	1.79	1.88	2.01

Panel B: CDS spreads (bps)

	Maturity (years)							
	1	2	3	5	7	10	15	20
AAA	3	5	8	11	13	14	15	17
s.e.	3	3	3	4	5	5	5	5
AA	1	3	5	9	13	18	24	29
s.e.	1	2	3	4	5	6	7	7
A	1	6	11	21	30	40	53	62
s.e.	1	3	6	9	10	11	12	12
BBB	8	24	39	62	79	96	114	124
s.e.	4	10	14	17	18	18	18	18
BB	62	117	152	193	214	230	242	248
s.e.	17	28	32	33	33	32	31	30
B	220	333	396	455	476	485	486	483
s.e.	52	56	54	49	45	42	39	38
C	677	946	1065	1112	1100	1076	1048	1032
s.e.	107	111	100	85	80	77	75	73

Table A2

Cumulative default rates (%) in the data for the sample period 1940−2017 and the Black-Cox model. This table reports the cumulative default rates across credit ratings and horizons in the data and the Black-Cox model. The data (Panel A) are the Moody's average historical default rates from 1940 to 2017. Panel B reports the default rates predicted by the Black-Cox model when asset value is set to the sum of market equity and book debt. We get a default boundary $d = 0.64$ when minimizing the horizon-weighted average absolute error of default rates from FS. Panel C reports the default rates predicted by the Black-Cox model when asset value is set to the sum of market equity and market debt. We get a default boundary $d = 0.55$ in this case. Standard errors (s.e.) are computed using the bootstrap method.

	Horizon (years)										
	1	2	3	4	5	6	8	10	12	15	20
Panel A: Historical default rate (1940-2017)											
AAA	0.00	0.02	0.05	0.07	0.13	0.19	0.34	0.54	0.66	0.88	1.18
AA	0.05	0.12	0.20	0.31	0.42	0.55	0.79	0.95	1.05	1.27	1.94
A	0.04	0.11	0.23	0.35	0.48	0.64	0.94	1.21	1.42	1.76	2.51
BBB	0.14	0.37	0.65	0.99	1.33	1.64	2.31	3.04	3.75	4.89	6.49
BB	0.75	1.86	2.98	4.11	5.22	6.22	7.94	9.52	10.68	12.25	14.54
B	3.00	5.88	8.60	10.85	12.73	14.45	17.26	19.43	20.88	23.47	25.56
C	11.89	17.30	20.57	23.16	25.16	26.99	30.51	32.72	33.87	35.66	36.82
Panel B: Model using book value of debt ($d = 0.64$, $s.e. = 0.05$)											
AAA	0.00	0.04	0.13	0.23	0.35	0.46	0.67	0.84	1.00	1.19	1.52
s.e.	0.00	0.02	0.05	0.07	0.10	0.14	0.20	0.25	0.30	0.36	0.43
AA	0.00	0.04	0.09	0.16	0.25	0.35	0.59	0.85	1.12	1.52	2.26
s.e.	0.00	0.03	0.05	0.07	0.10	0.12	0.17	0.22	0.27	0.35	0.47
A	0.01	0.06	0.19	0.37	0.59	0.84	1.37	1.93	2.52	3.43	5.07
s.e.	0.00	0.04	0.09	0.16	0.23	0.29	0.41	0.52	0.61	0.71	0.84
BBB	0.04	0.30	0.71	1.23	1.79	2.39	3.60	4.82	6.01	7.73	10.52
s.e.	0.02	0.11	0.22	0.32	0.40	0.47	0.59	0.68	0.76	0.86	1.01
BB	0.43	1.61	2.98	4.38	5.74	7.03	9.38	11.46	13.30	15.72	19.23
s.e.	0.09	0.28	0.44	0.55	0.63	0.70	0.81	0.92	1.02	1.15	1.35
B	1.47	4.29	7.27	10.14	12.77	15.15	19.23	22.59	25.38	28.83	33.45
s.e.	0.24	0.44	0.51	0.56	0.60	0.64	0.69	0.71	0.72	0.74	0.80
C	1.55	5.46	9.65	13.57	17.07	20.17	25.47	29.95	33.75	38.34	43.78
s.e.	0.40	0.92	1.39	1.76	2.05	2.29	2.75	3.06	3.10	2.88	2.50
Panel C: Model using market value of debt ($d = 0.55$, $s.e. = 0.04$)											
AAA	0.00	0.01	0.03	0.06	0.11	0.16	0.28	0.38	0.48	0.62	0.84
s.e.	0.00	0.00	0.01	0.03	0.04	0.06	0.10	0.13	0.16	0.21	0.26
AA	0.00	0.01	0.03	0.06	0.11	0.16	0.31	0.48	0.66	0.96	1.51
s.e.	0.00	0.01	0.02	0.04	0.06	0.08	0.13	0.18	0.23	0.31	0.43
A	0.00	0.03	0.11	0.23	0.37	0.55	0.94	1.37	1.82	2.54	3.87
s.e.	0.00	0.02	0.06	0.12	0.18	0.24	0.36	0.47	0.57	0.68	0.84
BBB	0.02	0.16	0.43	0.78	1.18	1.62	2.55	3.52	4.49	5.94	8.36
s.e.	0.01	0.07	0.17	0.27	0.36	0.44	0.58	0.69	0.78	0.89	1.04
BB	0.21	0.96	1.92	2.95	3.98	5.00	6.93	8.70	10.32	12.50	15.77
s.e.	0.05	0.23	0.41	0.56	0.68	0.78	0.93	1.04	1.14	1.26	1.44
B	0.81	2.64	4.72	6.84	8.88	10.82	14.34	17.40	20.04	23.41	28.05
s.e.	0.19	0.48	0.64	0.72	0.75	0.76	0.76	0.74	0.73	0.73	0.78
C	3.13	10.02	17.57	24.60	30.48	35.05	41.47	45.91	49.23	52.99	57.59
s.e.	0.97	2.71	4.09	4.52	4.19	3.70	3.10	2.79	2.59	2.41	2.26

Table A3
CDS spreads (bps) in the data and the Black-Cox model. This table reports the average CDS spreads across credit ratings and maturities in the data and the Black-Cox model. The data (Panel A) report the average CDS spreads in our sample (2002−2017). Panel B reports the average CDS spreads predicted by the Black-Cox model when we set asset value to be the sum of market equity and book debt and the default boundary $d = 0.64$ from Panel B in Table A2. Panel C reports the average CDS spreads predicted by the Black-Cox model when we set asset value to be the sum of market equity and market debt and the default boundary $d = 0.55$ from Panel C in Table A2. Standard errors (s.e.) are computed using the bootstrap method.

	Maturity (years)							
	1	2	3	5	7	10	15	20
	Panel A: Data							
AAA	30	32	33	37	41	45	47	48
AA	21	26	31	40	47	54	59	61
A	20	26	33	48	58	68	74	77
BBB	53	66	79	108	124	137	144	148
BB	156	187	216	271	292	303	308	310
B	351	403	449	531	550	551	550	549
C	950	1037	1098	1164	1143	1119	1111	1095
	Panel B: Model using book value of debt ($d = 0.64$, $s.e. = 0.05$)							
AAA	3	4	6	9	11	12	14	15
s.e.	2	2	3	3	4	4	5	5
AA	1	2	4	7	11	16	21	26
s.e.	0	1	2	3	3	4	4	5
A	1	4	8	17	25	34	46	55
s.e.	0	2	4	6	7	8	8	8
BBB	5	17	29	50	65	81	99	109
s.e.	2	6	8	10	10	10	10	10
BB	40	85	116	154	175	193	208	215
s.e.	8	14	16	16	15	15	14	14
B	146	235	291	352	379	396	404	406
s.e.	26	26	24	21	19	17	15	15
C	169	283	352	420	448	466	473	471
s.e.	41	47	50	51	51	49	44	41
	Panel C: Model using market value of debt ($d = 0.55$, $s.e. = 0.04$)							
AAA	1	2	2	4	5	6	7	9
s.e.	1	1	1	1	2	2	3	3
AA	0	1	1	3	6	9	14	18
s.e.	0	0	1	2	2	3	4	4
A	0	2	5	11	17	24	34	43
s.e.	0	1	3	5	6	7	8	8
BBB	2	9	18	33	46	60	76	87
s.e.	1	4	7	9	10	10	10	10
BB	20	51	75	107	127	145	162	171
s.e.	5	12	15	17	17	16	16	15
B	84	147	190	245	275	299	314	320
s.e.	20	25	25	21	19	17	16	15
C	296	496	624	746	775	781	775	771
s.e.	81	127	144	127	115	109	104	102

Table A4

Cross-sectional and time-series variations of the jump-diffusion structural model implied default boundary. This table reports the estimation results of the default boundary as a function of a cross-sectional variable or a time-series aggregate variable in the jump-diffusion structural model. To perform this estimation, we use the functional form for the scaled default boundary $d_{i,t} = 1/\left(1 + e^{-A-BX_{i,t}}\right)$. Panel A reports the results for two firm-level variables: i) distance-to-default (DD) and ii) credit ratings. Panel B reports the results for five time-series variables: i) average BBB-AAA credit spread, ii) the cyclically adjusted price-to-earnings ratio (CAPE), iii) the one-year risk-free rate, iv) aggregate leverage ratio computed as the cross-sectional mean of leverage ratio measured by the ratio of book debt and the sum of book debt and market equity, and v) the cross-sectional mean of asset volatility (σ_i) implied by the jump-diffusion structural model. Standard errors (s.e.) are computed using the bootstrap method.

	Panel A: Cross-section			Panel B: Time-series	
	Est.	s.e.		Est.	s.e.
	X = DD			X = BBB-AAA	
A	0.49	0.19	A	0.61	0.04
B	0.03	0.03	B	0.00	0.03
	X = Credit rating			X = CAPE	
A	1.00	0.68	A	0.53	0.19
B	-0.08	0.15	B	0.00	0.01
				$X = R_f$ (1 Year)	
			A	0.57	0.06
			B	0.01	0.01
				X = Mean BD/(BD+ME)	
			A	0.63	0.06
			B	-0.06	0.19
				X = Mean σ_i	
			A	0.61	0.04
			B	0.02	0.14

Table A5

Pricing errors of the jump-diffusion structural model across parameters d and λ^Q. This table reports the sample average pricing error as a function of the default boundary d and the risk-neutral jump intensity λ^Q that are assumed to be constants for the entire sample. All other parameters are optimized using the corresponding error functions. Panel A reports the average RMSE of the CDS, which is the error function used to estimate the risk-neutral parameters in the main text of the paper. Panel B reports the total squared error including both the risk-neutral and real RMSE, which is the error function of a joint estimation of the risk-neural and real parameters in a single step. In both estimations, we get the estimated default boundary $d = 0.65$ and the risk-neutral jump intensity $\lambda^Q = 0.2$.

Panel A: Average risk-neutral pricing errors (RMSE)							
$d \backslash \lambda^Q$	0.17	0.18	0.19	0.2	0.21	0.22	0.23
0.63		12.793		12.768		12.830	
0.64			12.774	12.744	12.778		
0.65	12.833		12.760	12.742	12.784		12.885
0.66			12.761	12.754	12.789		
0.67		12.764		12.764		12.814	

Panel B: Total errors (risk-neutral and real RMSE)							
$d \backslash \lambda^Q$	0.17	0.18	0.19	0.2	0.21	0.22	0.23
0.63		172.67		171.91		173.50	
0.64			172.03	171.26	172.12		
0.65	173.58		171.57	171.15	172.14		174.72
0.66			171.55	171.33	172.21		
0.67		171.54		171.63		172.78	

Table A6
The CDS spread differences (bps) between the data and the models. This table reports the CDS spread differences (bps) between i) the data and the jump-diffusion structural model and ii) the data and the diffusion-based model. We compute the mean/median differences for each credit rating and maturity. Panel A reports, for each credit rating, the differences in means averaging over all the maturities. Panel B reports, for each credit rating, the differences in medians averaging over all the maturities. Standard errors (s.e.) are computed using the bootstrap method.

	Panel A: Mean		Panel B: Median	
	Jump-diffusion	Diffusion-based	Jump-diffusion	Diffusion-based
AAA	0.44	14.31	0.49	14.25
s.e.	0.26	3.21	0.32	4.16
AA	0.21	16.41	0.02	14.97
s.e.	0.06	3.26	0.30	3.23
A	0.14	21.62	0.15	20.71
s.e.	0.04	4.01	0.13	4.13
BBB	0.45	35.02	0.06	32.45
s.e.	0.25	7.48	0.21	6.49
BB	4.17	55.69	0.33	53.01
s.e.	0.98	12.52	0.67	11.28
B	18.84	74.87	15.17	75.86
s.e.	2.68	15.28	3.82	14.59
C	57.36	74.60	21.57	65.14
s.e.	10.35	16.21	9.50	19.04