

The Internet Appendix for
The Leverage Effect and the Basket-Index Put Spread

A.1. A model with stochastic volatility and stochastic correlation

In the text we demonstrated that both the diffusion-based model of Merton (1974) and the jump-diffusion model of Merton (1976) (modified to exhibit a leverage effect) capture well the time series of basket-index put spreads for all sectors during both the crisis and precrisis periods. One possible concern with these models, however, is that they assume that both asset volatility and asset correlation are constant in time, which is strongly contradicted by the data. For example, Fig. A.1 shows the time series of cumulative asset returns, volatility and correlation for the banking sector. Clearly, the drop in asset prices during the financial crisis was associated with increases in both asset volatility and asset correlation. In order to demonstrate that our findings are robust to these empirical facts, here we investigate a model that captures both stochastic volatility and stochastic correlation.

To maintain tractability and remain relatively close to the models of KLN, we investigate a model similar to that proposed by Heston (1993) (modified to exhibit a leverage effect). In particular, we assume that returns on the aggregate index ($\frac{dA_t}{A_t}$) and on (ex-ante identical) individual firms ($\frac{dA_{i,t}}{A_{i,t}}$) follow:

$$\begin{aligned}
 \frac{dA_t}{A_t} &= (r - \delta) dt + \sigma_{m1} dz_{m1} + \sqrt{V_t} dz_{m2} \\
 dV &= \kappa (\bar{V} - V_t) dt - \alpha_2 \sqrt{V_t} dz_{m2} - \alpha_3 \sqrt{V_t} dz_{m3} \\
 \frac{dA_{i,t}}{A_{i,t}} &= \frac{dA_t}{A_t} + \sigma_i dz_i,
 \end{aligned} \tag{A.1}$$

where, as above, the law of large numbers can be used to show that

$$A_t = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N A_{i,t}. \tag{A.2}$$

Specifying $\alpha_2 > 0$ implies that innovations in returns and variance are negatively correlated. Moreover, note that the variance, covariance, and correlation are stochastic and increasing in V_t :

$$\begin{aligned}
\text{Var}(dA_{i,t}) &\equiv \frac{1}{dt} \left(\frac{dA_{i,t}}{A_{i,t}} \right)^2 = \sigma_{m1}^2 + V_t + \sigma_i^2 \\
\text{Cov}(dA_{i,t}, dA_{j,t}) &\equiv \frac{1}{dt} \left(\frac{dA_{i,t}}{A_{i,t}} \right) \left(\frac{dA_{j,t}}{A_{j,t}} \right) = \sigma_{m1}^2 + V_t \\
\text{Corr}(dA_{i,t}, dA_{j,t}) &\equiv \frac{\text{Cov}(dA_{i,t}, dA_{j,t})}{\text{Var}(dA_{i,t})} = \left(\frac{\sigma_{m1}^2 + V_t}{\sigma_{m1}^2 + V_t + \sigma_i^2} \right). \tag{A.3}
\end{aligned}$$

Therefore, variance, covariance and correlation are also negatively correlated with asset returns, consistent with the empirical evidence noted above.

The rest of the analysis is a straightforward generalization of results found in the text. The date-0 stock price is

$$\begin{aligned}
S_{i,0} &= e^{-rT} \mathbb{E}_0^Q [\max(0, A_{i,T} - D)] \\
&= e^{-rT} \int_0^\infty dA_T \pi^Q(A_T | A_0, V_0) A_T N \left(\frac{\log \left(\frac{A_T}{D} \right) + \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right) \\
&\quad - D e^{-rT} \int_0^\infty dA_T \pi^Q(A_T | A_0, V_0) N \left(\frac{\log \left(\frac{A_T}{D} \right) - \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right). \tag{A.4}
\end{aligned}$$

By using the identity (or a similar identity with change of variables ($x \Rightarrow -x$)):

$$N \left(\frac{\bar{z}}{\sqrt{\sigma_x^2 + \sigma_y^2}} \right) = \int_{-\infty}^\infty dx \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{x^2}{2\sigma_x^2}} N \left(\frac{\bar{z} - x}{\sigma_y} \right), \tag{A.5}$$

it is straightforward to show that in the limit of constant volatility (e.g., $\kappa = 0$ and $\alpha_2 = \alpha_3 = 0$) Eq. (A.4) reduces to Eq. (??) in the text.

Similarly, in analogy with Eq. (??) in the text, we find that the date-0 put option price is:

$$\begin{aligned}
P_{i,0} &= e^{-rT} \mathbf{E}^Q [\max(0, K - S_{i,T})] \\
&= (K + D) e^{-rT} \int_0^\infty dA_T \pi^Q(A_T | A_0, V_0) N \left(\frac{-\log \left(\frac{A_T}{D+K} \right) + \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right) \\
&\quad - D e^{-rT} \int_0^\infty dA_T \pi^Q(A_T | A_0, V_0) N \left(\frac{-\log \left(\frac{A_T}{D} \right) + \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right) \\
&\quad - e^{-rT} \int_0^\infty dA_T \pi^Q(A_T | A_0, V_0) A_T \left[N \left(\frac{-\log \left(\frac{A_T}{D+K} \right) - \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right) - N \left(\frac{-\log \left(\frac{A_T}{D} \right) - \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right) \right].
\end{aligned} \tag{A.6}$$

Moreover, in analogy with Eq. (??), the price of a call option is

$$\begin{aligned}
C_{i,0} &= e^{-rT} \mathbf{E}_0^Q [\max(0, S_{i,T} - K)] \\
&= e^{-rT} \int_0^\infty dA_T \pi^Q(A_T | A_0, V_0) A_T N \left(\frac{\log \left(\frac{A_T}{D+K} \right) + \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right) \\
&\quad - (K + D) e^{-rT} \int_0^\infty dA_T \pi^Q(A_T | A_0, V_0) N \left(\frac{\log \left(\frac{A_T}{D+K} \right) - \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right).
\end{aligned} \tag{A.7}$$

In analogy with Eq. (??), the final index price conditional upon A_T is

$$S_T(A_T) = A_T N \left(\frac{\log \left(\frac{A_T}{D} \right) + \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right) - D N \left(\frac{\log \left(\frac{A_T}{D} \right) - \frac{\sigma_i^2}{2} T}{\sqrt{\sigma_i^2 T}} \right). \tag{A.8}$$

As such, the date-0 price of a put option written on this index with strike price K and maturity T can be determined via:

$$\begin{aligned}
P_0 &= e^{-rT} \mathbf{E}_0^Q [\max[0, K - S_T(A_T)]] \\
&= e^{-rT} \int_0^\infty dA_T \pi^Q(A_T | A_0, V_0) \max[0, K - S_T(A_T)],
\end{aligned} \tag{A.9}$$

whereas the date-0 price of a call option written on this index with strike price K and maturity

T can be determined via:

$$\begin{aligned} C_0 &= e^{-rT} \mathbb{E}_0^Q \left[\max [0, S_T (A_T) - K] \right] \\ &= e^{-rT} \int_0^\infty dA_T \pi^Q(A_T | A_0, V_0) \max [0, S_T (A_T) - K]. \end{aligned} \quad (\text{A.10})$$

To solve for prices, we need the transition density $\pi^Q(A_T | A_0, V_0)$. To determine this transition density, we first apply Ito's lemma to $a_t \equiv \log A_t$ to obtain

$$da = \left(\mu^Q - \frac{1}{2} V_t \right) dt + \sigma_{m1} dz_{m1} + \sqrt{V_t} dz_{m2}, \quad (\text{A.11})$$

where we have defined $\mu^Q \equiv (r - \delta - \frac{\sigma_{m1}^2}{2})$. Then, following Heston (1993), we introduce the characteristic function:

$$\begin{aligned} G^\lambda(a_t, V_t, t) &\equiv \mathbb{E}_t^Q [e^{i\lambda a_T}] \\ &= \int_{-\infty}^\infty da_T \pi^Q(A_T | A_0, V_0) e^{i\lambda a_T}. \end{aligned} \quad (\text{A.12})$$

Because $G^\lambda(a_t, V_t, t)$ is a Q-martingale, it follows that

$$0 = G_t + G_a \left(\mu^Q - \frac{1}{2} V_t \right) + G_V \kappa (\bar{V} - V) + \frac{1}{2} G_{aa} (\sigma_{m1}^2 + V) + \frac{1}{2} G_{VV} (\alpha_2^2 + \alpha_3^2) V + G_{aV} (-\alpha_2 V). \quad (\text{A.13})$$

Given that the dynamics are affine, it follows from Duffie and Kan (1996) that the solution is of the form:

$$G^\lambda(a_t, V_t, t) = e^{D(T-t) + B(T-t)a_t + C(T-t)V_t}, \quad (\text{A.14})$$

Plugging Eq. (A.14) into Eq. (A.13), defining $\tau \equiv (T - t)$, and collecting terms linear in V , linear in a , and independent of (V, a) generates the coupled system of ODE's:

$$\begin{aligned} C' &= -\frac{1}{2} B - C\kappa + \frac{1}{2} B^2 + \left(\frac{\alpha_2^2 + \alpha_3^2}{2} \right) C^2 - \alpha_2 BC \\ B' &= 0 \\ D' &= \mu^Q B + \kappa \bar{V} C + \frac{\sigma_{m1}^2}{2} B^2, \end{aligned} \quad (\text{A.15})$$

subject to the initial conditions $D(\tau = 0) = 0$, $B(\tau = 0) = i\lambda$, $C(\tau = 0) = 0$. It is convenient to define $\psi \equiv \frac{1}{2}(\omega_1 - \omega_2) (\alpha_2^2 + \alpha_3^2)$. The solutions are:

$$\begin{aligned}
B(\tau) &= i\lambda \\
C(\tau) &= \omega_1\omega_2 \left(\frac{1 - e^{\psi\tau}}{\omega_2 - \omega_1 e^{\psi\tau}} \right) \\
D(\tau) &= \left(i\lambda\mu^Q - \frac{\lambda^2\sigma_{m1}^2}{2} \right) \tau + \left(\frac{\kappa\bar{V}\omega_1}{\psi} \right) \left[\psi\tau - \log \left(\frac{\omega_2 - \omega_1 e^{\psi\tau}}{\omega_2 - \omega_1} \right) \right] \\
&\quad + \left(\frac{\kappa\bar{V}\omega_2}{\psi} \right) \log \left(\frac{\omega_2 - \omega_1 e^{\psi\tau}}{\omega_2 - \omega_1} \right),
\end{aligned} \tag{A.16}$$

where (ω_1, ω_2) satisfy

$$0 = \omega^2 - \left(\frac{2(\kappa + i\alpha_2\lambda)}{(\alpha_2^2 + \alpha_3^2)} \right) \omega - \left(\frac{1}{(\alpha_2^2 + \alpha_3^2)} \right) (\lambda^2 + i\lambda). \tag{A.17}$$

We use the Fourier Transform to solve the risk-neutral distribution of asset A_T from its characteristic function. Then, we use the distribution to compute the risk-neutral expected payoffs of the options to price them.

Table A.1 reports the parameters and the average option prices predicted by the model. Panel A reports the parameters of the model. For each date, we set asset variance V_t , the level of debt obligation D , and idiosyncratic asset volatility σ_i to match the basket call, initial leverage, and asset correlation, respectively. We set $\alpha_2 = 0.025$ and $\alpha_3 = 0.08$ to match the realized volatility of V_t and asset return-variance correlation ($\text{Cov}(A, V)$) in the data. $\bar{V} = 0.0032$ is set to match the time series average of V_t implied by the model. We set $\kappa = 0.2$ to match the average basket-index put spread in the precrisis period. Panel B reports the model predicted option prices. The model generates an average basket-index put spread 4.4 cents per dollar during the crisis period, matching the results from the MER74-H and the jump-diffusion models, and closely matching the data (4.6 cents per dollar).

A.2. A jump-diffusion model

Here, we provide additional details for the jump-diffusion model introduced in Section ???. Our model is based on Merton (1976) (which KLN investigate), although we modify it to account for a leverage effect. Log-consumption $c_t = \log C_t$ is exogenously specified as a jump-diffusion process under

the statistical P-measure:

$$dc = g dt + \sqrt{\rho}\sigma dz + \eta dq - \theta\lambda dt, \quad (\text{A.18})$$

where the jump intensity is $(\frac{1}{dt}) \mathbb{E}[dq] = \lambda$, and the stochastic jump size is distributed normally $\eta \overset{P}{\sim} N(\theta, \nu^2)$. The preferences of the representative agent are specified to be constant relative risk aversion (CRRA), implying that marginal utility can be expressed as

$$\Lambda \equiv u_c = e^{-\zeta t - \alpha c}, \quad (\text{A.19})$$

where α is the relative risk aversion parameter, and ζ is the time preference parameter. Using Ito's lemma extended for jumps, it follows that the stochastic discount factor for this model is:

$$\frac{d\Lambda}{\Lambda} = -r dt - \alpha\sqrt{\rho}\sigma dz + dq(e^{-\alpha\eta} - 1) - \lambda dt \left(e^{-\alpha\theta + \frac{\alpha^2\nu^2}{2}} - 1 \right), \quad (\text{A.20})$$

where the risk free rate r is a constant. Defining the risk-neutral intensity $\lambda^Q \equiv (\frac{1}{dt}) \mathbb{E}^Q[dq]$, and the risk-neutral distribution jump parameters $\eta \overset{Q}{\sim} (\mu^Q, \nu_Q^2)$, standard change-of-measure calculations imply that:

$$\begin{aligned} dz &= dz^Q - \alpha\sigma\sqrt{\rho} dt \\ \lambda^Q &= \lambda e^{-\alpha\theta + \frac{\alpha^2\nu^2}{2}} \\ \mu^Q &= \theta - \alpha\nu^2 \\ \nu_Q &= \nu. \end{aligned} \quad (\text{A.21})$$

Because the agent faces a time-invariant investment opportunity set, this economy generates a constant consumption-wealth (i.e., payout) ratio, which we define as δ . Standard calculations imply that asset dynamics under the risk neutral measure follow:

$$\frac{dA_t}{A_t} + \delta dt = r dt + \sqrt{\rho}\sigma dz^Q + (e^\eta - 1) dq - \bar{\mu}^Q \lambda^Q dt, \quad (\text{A.22})$$

where $\bar{\mu}^Q \equiv \mathbb{E}^Q[e^\eta - 1] = \exp\left(\mu^Q + \frac{\nu^2}{2}\right) - 1$.

Following KLN, we specify firm- i dynamics by adding to aggregate asset dynamics both idiosyncratic

volatility and idiosyncratic jumps:¹

$$\begin{aligned} \frac{dA_{i,t}}{A_{i,t}} + \delta dt &= r dt + \sqrt{\rho} \sigma dz^Q + \sqrt{1-\rho} \sigma dz_i^Q \\ &+ (e^\eta - 1) dq - \bar{\mu}^Q \lambda^Q dt + (e^{\eta_i} - 1) dq_i - \bar{\mu}_i^Q \lambda_i^Q dt, \end{aligned} \quad (\text{A.23})$$

where, following KLN, we specify the idiosyncratic jump size η_i to have the same P-distribution as the aggregate jumps: $\eta_i \stackrel{P}{\sim} N(\theta, \nu^2)$.² Because idiosyncratic shocks do not command a risk premia, it follows that idiosyncratic jump size has the same distribution under the risk-neutral measure: $\eta_i \stackrel{Q}{\sim} N(\theta, \nu^2)$. For the same reason, the risk-neutral jump intensity equals the physical intensity: $\lambda_i^Q = \lambda_i$. As above, we find it convenient to define the constant $\bar{\mu}_i^Q = E^Q [e^{\eta_i} - 1] = \exp\left(\theta + \frac{\nu^2}{2}\right) - 1$.

From Ito's lemma, log-firm dynamics follow:

$$d \log A_{i,t} = \left[r - \delta - \frac{1}{2} \sigma^2 - \bar{\mu}^Q \lambda^Q - \bar{\mu}_i^Q \lambda_i^Q \right] dt + \sqrt{\rho} \sigma dz^Q + \sqrt{1-\rho} \sigma dz_i^Q + \eta dq + \eta_i dq_i. \quad (\text{A.24})$$

Following KLN, in order to guarantee that our model is calibrated to historical correlation, we set jump correlation equal to Brownian motion correlation (ρ). Further, mostly following KLN, we specify jump intensities to be an increasing function of the parameter controlling variance from Brownian motions (σ^2). Based on these restrictions, we specify aggregate and firm-specific jump intensities as:

$$\begin{aligned} \lambda &= \rho (\omega \sigma^2 + \lambda_0) \\ \lambda_i &= (1 - \rho) (\omega \sigma^2 + \lambda_0), \end{aligned} \quad (\text{A.25})$$

where ω and λ_0 are free parameters.

We solve this jump-diffusion model using the method of Merton (1976). Conditional on ($dq = j$) sector-wide jumps and ($dq_i = k$) idiosyncratic jumps up until T , log asset value at T follows

$$\begin{aligned} \log A_{i,T}(j, k) - \log A_{i,0} &= \left[r - \delta - \frac{1}{2} \sigma^2 - \bar{\mu}^Q \lambda^Q - \bar{\mu}_i^Q \lambda_i^Q \right] T + j \mu^Q + k \theta \\ &+ \sqrt{T \rho \sigma^2 + j \nu^2} u + \sqrt{T(1-\rho)\sigma^2 + k \nu^2} u_i, \end{aligned} \quad (\text{A.26})$$

¹In the limit of an infinite number of identical firms, equations (A.23) and (A.22) are consistent with $A_t = \lim_{N \Rightarrow \infty} \frac{1}{N} \sum_{i=1}^N A_{i,t}$.

²This specification for the random jump size, combined with the specifications of the jump intensities in Eq. (A.25), guarantees that asset return correlation is controlled by a single parameter ρ .

where u represents a sector-wide standard normal shock (which includes both sector-wide diffusion risk and sector-wide jump risk): $u \stackrel{Q}{\sim} N(0, 1)$. Similarly, u_i represents an idiosyncratic standard normal shock (which includes both idiosyncratic diffusion risk and idiosyncratic jump risk): $u_i \stackrel{Q}{\sim} N(0, 1)$. The total conditional log asset volatility at T is thus $\sqrt{T\sigma^2 + (j+k)\nu^2}$.

A.2.1. Equity and equity options

Conditional on the numbers of jumps (j, k) , asset dynamics from dates $t \in (0, T)$ can be expressed as a diffusion process:

$$\frac{dA_{i,t}(j, k)}{A_{i,t}(j, k)} = \mu_{j,k} dt + \sigma_{j,k} dz^Q, \quad (\text{A.27})$$

where

$$\mu_{j,k} = r - \delta - \frac{\sigma^2}{2} - \bar{\mu}^Q \lambda^Q - \bar{\mu}_i^Q \lambda_i^Q + \frac{(j\mu^Q + k\theta)}{T} + \frac{1}{2}\sigma_{j,k}^2 \quad (\text{A.28})$$

$$\sigma_{j,k} = \sqrt{\sigma^2 + \frac{(j+k)\nu^2}{T}}. \quad (\text{A.29})$$

Therefore, in analogy with Eq. (??), equity value conditional on state (j, k) is

$$S_{i,0}(j, k) = A_{i,0} e^{(\mu_{j,k} - r)T} N\left(\frac{\log\left(\frac{A_{i,0}}{D}\right) + (\mu_{j,k} + \frac{\sigma_{j,k}^2}{2})T}{\sqrt{\sigma_{j,k}^2 T}}\right) - D e^{-rT} N\left(\frac{\log\left(\frac{A_{i,0}}{D}\right) + (\mu_{j,k} - \frac{\sigma_{j,k}^2}{2})T}{\sqrt{\sigma_{j,k}^2 T}}\right). \quad (\text{A.30})$$

With constant jump intensity λ^Q , the number of jumps in a period T follows a Poisson distribution. As such, the date-0 equity value is the probability-weighted average:

$$S_{i,0} = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \left(\frac{e^{-\lambda^Q T} (\lambda^Q T)^j}{j!}\right) \left(\frac{e^{-\lambda_i^Q T} (\lambda_i^Q T)^k}{k!}\right) S_{i,0}(j, k). \quad (\text{A.31})$$

We can price European options written on equity using the same approach. For example, the

conditional value of a put option written on equity is

$$\begin{aligned}
P_{i,0}(j, k) = & (K + D)e^{-rT} N \left(\frac{-\log \left(\frac{A_{i,0}}{D+K} \right) - (\mu_{j,k} - \frac{\sigma_{j,k}^2}{2})T}{\sqrt{\sigma_{j,k}^2 T}} \right) \\
& - De^{-rT} N \left(\frac{-\log \left(\frac{A_{i,0}}{D} \right) - (\mu_{j,k} - \frac{\sigma_{j,k}^2}{2})T}{\sqrt{\sigma_{j,k}^2 T}} \right) \\
& - A_{i,0} e^{(\mu_{j,k} - r)T} \left[N \left(\frac{-\log \left(\frac{A_{i,0}}{D+K} \right) - (\mu_{j,k} + \frac{\sigma_{j,k}^2}{2})T}{\sqrt{\sigma_{j,k}^2 T}} \right) - N \left(\frac{-\log \left(\frac{A_{i,0}}{D} \right) - (\mu_{j,k} + \frac{\sigma_{j,k}^2}{2})T}{\sqrt{\sigma_{j,k}^2 T}} \right) \right].
\end{aligned} \tag{A.32}$$

Similarly, the conditional value of a call option written on equity is

$$\begin{aligned}
C_{i,0}(j, k) = & A_{i,0} e^{(\mu_{j,k} - r)T} N \left(\frac{\log \left(\frac{A_{i,0}}{D+K} \right) + (\mu_{j,k} + \frac{\sigma_{j,k}^2}{2})T}{\sqrt{\sigma_{j,k}^2 T}} \right) \\
& - (K + D)e^{-rT} N \left(\frac{\log \left(\frac{A_{i,0}}{D+K} \right) + (\mu_{j,k} - \frac{\sigma_{j,k}^2}{2})T}{\sqrt{\sigma_{j,k}^2 T}} \right).
\end{aligned} \tag{A.33}$$

We then determine put and call option prices following the same approach used to price the stock in Eq. (A.31):

$$\begin{aligned}
P_{i,0} &= \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \left(\frac{e^{-\lambda^Q T} (\lambda^Q T)^j}{j!} \right) \left(\frac{e^{-\lambda_i^Q T} (\lambda_i^Q T)^k}{k!} \right) P_{i,0}(j, k) \\
C_{i,0} &= \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \left(\frac{e^{-\lambda^Q T} (\lambda^Q T)^j}{j!} \right) \left(\frac{e^{-\lambda_i^Q T} (\lambda_i^Q T)^k}{k!} \right) C_{i,0}(j, k).
\end{aligned} \tag{A.34}$$

A.2.2. Index and index options

Conditional on the numbers of jumps (j, k) and the sector-wide shock u , asset dynamics collapse to the diffusion process:

$$\frac{dA_{i,t}(j, k, u)}{A_{i,t}(j, k, u)} = \mu_{j,k,u} dt + \sigma_{j,k,u} dz_i^Q, \tag{A.35}$$

where

$$\mu_{j,k,u} = r - \delta - \frac{\sigma^2}{2} - \bar{\mu}^Q \lambda^Q - \bar{\mu}_i^Q \lambda_i^Q + \frac{j\mu^Q + k\theta + \sqrt{T\rho\sigma^2 + j\nu^2} u}{T} + \frac{1}{2}\sigma_{j,k,u}^2 \quad (\text{A.36})$$

$$\sigma_{j,k,u} = \sqrt{(1 - \rho)\sigma^2 + \frac{k\nu^2}{T}}. \quad (\text{A.37})$$

In analogy to equation (??) in the text, the conditional payoff of a stock index at T is

$$S_{i,T}(j, k, u) = A_{i,0} e^{\mu_{j,k,u} T} N \left(\frac{\log \left(\frac{A_{i,0}}{D} \right) + (\mu_{j,k,u} + \frac{\sigma_{j,k,u}^2}{2}) T}{\sqrt{\sigma_{j,k,u}^2 T}} \right) - DN \left(\frac{\log \left(\frac{A_{i,0}}{D} \right) + (\mu_{j,k,u} - \frac{\sigma_{j,k,u}^2}{2}) T}{\sqrt{\sigma_{j,k,u}^2 T}} \right). \quad (\text{A.38})$$

We can determine the date-0 price of the stock index using:

$$\begin{aligned} S_0 &= e^{-rT} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \left(\frac{e^{-\lambda^Q T} (\lambda^Q T)^j}{j!} \right) \left(\frac{e^{-\lambda_i T} (\lambda_i T)^k}{k!} \right) \int_{-\infty}^{\infty} S_{i,T}(j, k, u) \phi(u) du \\ &= \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \left(\frac{e^{-\lambda^Q T} (\lambda^Q T)^j}{j!} \right) \left(\frac{e^{-\lambda_i T} (\lambda_i T)^k}{k!} \right) S_{i,0}(j, k) \\ &= S_{i,0}, \end{aligned} \quad (\text{A.39})$$

where ϕ represents the standard normal density function. This equation captures the fact that all firms are specified to be ex-ante identical.

To determine the prices of options on the index, in analogy with Eq. (??), we define the date- T value of the index, conditional upon the sector sources of risk (j, u) :

$$\begin{aligned} S_T(j, u) &= \mathbf{E}^Q [S_{i,T}(j, k, u) | j, u] \\ &= \sum_{k=0}^{\infty} \left(\frac{e^{-\lambda^Q T} (\lambda^Q T)^k}{k!} \right) S_{i,T}(j, k, u). \end{aligned} \quad (\text{A.40})$$

The price of an index put can then be expressed as

$$P_0 = e^{-rT} \sum_{j=0}^{\infty} \frac{e^{-\lambda^Q T} (\lambda^Q T)^j}{j!} \int_{-\infty}^{\infty} \max[0, K - S_T(j, u)] \phi(u) du. \quad (\text{A.41})$$

Similarly, the price an index call can be determined via:

$$C_0 = e^{-rT} \sum_{j=0}^{\infty} \frac{e^{-\lambda^Q T} (\lambda^Q T)^j}{j!} \int_{-\infty}^{\infty} \max[0, S_T(j, u) - K] \phi(u) du. \quad (\text{A.42})$$

A.2.3. Credit spreads

Consider a defaultable bond issued by firm- i that at date- T pays \$1 if firm value is above D , and the recovery rate β if the date- T firm value falls below D :

$$B_{i,T} = \beta + (1 - \beta) \mathbf{1}_{(A_{i,T} > D)}. \quad (\text{A.43})$$

Conditional on the numbers of jumps (j, k) , its date-0 value is

$$\begin{aligned} B_{i,0}(j, k) &= e^{-rT} \mathbb{E}_0^Q \left[\beta + (1 - \beta) \mathbf{1}_{(A_{i,T} > D)} \mid j, k \right] \\ &= e^{-rT} \left[\beta + (1 - \beta) N \left(\frac{-\log(D/A_{i,0}) + \left(\mu_{j,k} - \frac{\sigma_{j,k}^2}{2} \right) T}{\sqrt{\sigma_{j,k}^2 T}} \right) \right]. \end{aligned} \quad (\text{A.44})$$

In analogy to the solution of the date-0 equity value, the date-0 bond value is

$$B_{i,0} = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \left(\frac{e^{-\lambda^Q T} (\lambda^Q T)^j}{j!} \right) \left(\frac{e^{-\lambda_i T} (\lambda_i T)^k}{k!} \right) B_{i,0}(j, k). \quad (\text{A.45})$$

Given the bond price $B_{i,0}$, we define the bond yield $y_{i,0}$ via $B_{i,0} \equiv e^{-y_{i,0} T}$. Its credit spread is the difference between the bond yield $y_{i,0}$ and the risk-free rate r .

A.3. Calibration using option-implied correlations

KLN calibrate their models using historical correlation. There are several concerns associated with this choice. First, correlations change over time, and because security prices are forward looking, it is preferable to have forward-looking estimates of correlations. Second, since this paper focuses on the financial crisis, during which multiple aggregate downward jumps occurred, it is possible that realized correlations were higher than ex-ante expected correlations over this time interval. Finally, if correlation-risk is priced, then physical measure and risk-neutral measure estimates of correlation

will differ, and risk-neutral estimates are needed to price securities.³ For all of these reasons, here we investigate the pricing implications of the MER74-H model calibrated to call-implied correlations rather than historical correlations. In particular, for each date, we choose ρ and σ to perfectly match basket call and index call prices. The model-predicted average option prices of financials are reported in Table A.2. We see that calibrating the model using call-implied correlations generates results very similar to those obtained when calibrating the model using historical correlations.

³See, for example, Driessen *et al.* (2009), Buraschi, Trojani, and Vedolin (2014b).

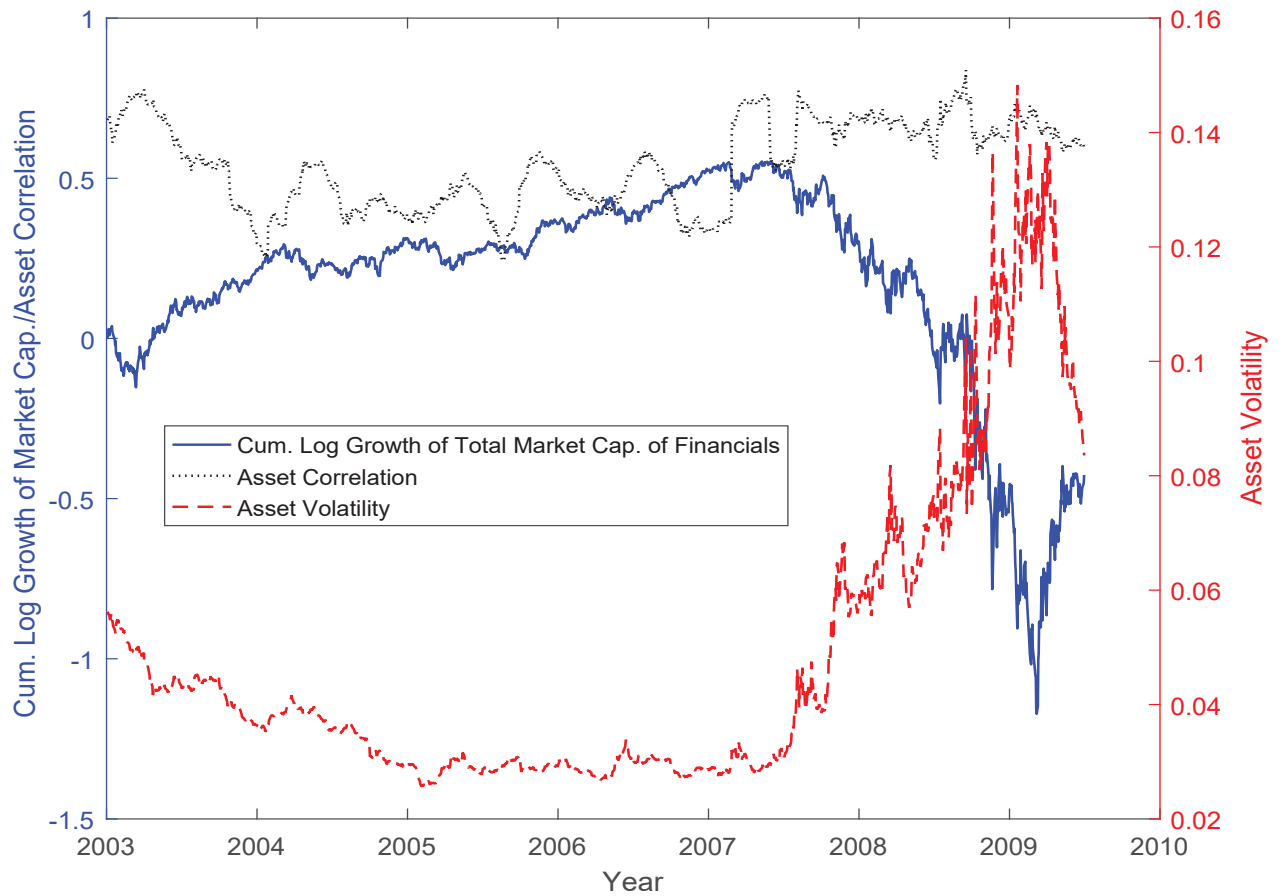


Fig. A.1. The time series of cumulative asset returns, asset correlation, and asset volatility for the financial sector. This figure reports three time series: i) cumulative asset return, defined as the cumulative log growth of total market equity of the financial sector; ii) realized asset correlation among firms; and iii) asset volatility implied by the MER74 model. The y-axis on the left-hand-side of the figure represents both the cumulative log growth of market equity and asset correlation. The y-axis on the right-hand-side of the figure represents asset volatility.

Table A.1

The stochastic volatility and correlation model. This table reports the parameters and average option prices of the stochastic volatility and correlation (SVC) model for financials during both the precrisis and crisis periods.

Panel A: Parameters						
Parameter			Value			
σ_{m1}			0.014			
κ			0.2			
\bar{V}			0.0032			
α_2			0.025			
α_3			0.08			

Panel B: Option prices						
	Puts			Calls		
	Basket	Index	Spread	Basket	Index	Spread
Data						
Precrisis	5.2	3.8	1.4	3.4	1.8	1.6
Crisis	15.9	11.3	4.6	5.5	3.4	2.1
SVC						
Precrisis	3.3	1.9	1.4	3.4	1.7	1.7
Crisis	14.4	9.9	4.4	5.5	3.1	2.4

Table A.2

Calibrating to call-implied correlation. This table reports average option prices for the financial sector during the precrisis and crisis periods for three cases: i) historical data, ii) predictions from the MER74-H model using historical data, and iii) predictions from the MER74-H model using forward looking call-implied correlations. Strike prices are from KLN, and correspond to index options with Black-Scholes implied deltas equal to ± 0.25 . Option prices are scaled by their strike prices.

	Puts			Calls		
	Basket	Index	Spread	Basket	Index	Spread
Data						
Precrisis	5.2	3.8	1.4	3.4	1.8	1.6
Crisis	15.9	11.3	4.6	5.5	3.4	2.1
MER74-H						
Precrisis	3.0	1.3	1.7	3.4	1.7	1.7
Crisis	13.9	9.5	4.4	5.5	3.5	2.1
Implied correlation						
Precrisis	3.0	1.4	1.6	3.4	1.8	1.6
Crisis	13.9	9.7	4.2	5.5	3.4	2.1