Online Appendix for: CoCo Futures as a Macroprudential and Microprudential Policy Tool

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I Introduction

This online Appendix is composed of three sections. Section 2 identifies cash flows and date-0 present values of bank equity and debt when banks issue *standard* futures contracts. Although the majority of the main text focuses on *CoCo* futures contracts, it also investigates the use of standard futures contracts in order to quantify the benefits associated with the bail-in feature of CoCo futures relative to standard futures, and this section provides some background information for that comparison. Section 3 illustrates some insights from Chowdry and Schwartz (2016), who identify a limitation of using standard futures contracts to hedge systematic risk when returns are not separable in systematic and idiosyncratic shocks. Section 4 extends the framework of Philippon and Wang (2021) by allowing banks to have access to futures contracts to hedge financial-sector risk. With this extended model, we demonstrate that all negative externalities associated with an underfunded banking system can be eliminated without any need of a bailout (or more accurately, with an arbitrarily small bailout). Moreover, the "performance-based bailout policy" policy tool of Philippon and Wang (2021) is completely supplanted once banks have access to futures contracts.

II GS Economy with Standard Futures Contract

Here we identify the bank's optimal capital structure and risk management policy function when it has access to a standard futures contract to hedge financial-sector risk. The results of this calculation are reported in Table IV of the main text in the column labeled, "Standard Futures."

For the case of a standard futures contract, we start with equation 36 of the main text, although we relabel the superscript appropriately. For this case, if positive, bank shareholders receive a dividend:

$$DIV_{B}^{STD}(\epsilon,\epsilon_{B}) \;=\; (1-\tau)\left[REV_{B}(\epsilon_{B}) + n_{H}\left(K-\hat{X}_{B}(\epsilon)\right) - DEP\, e^{\hat{r}T} - F_{B}\right] + \tau I_{B,E}. \quad (1)$$

We define the ϵ-dependent location of the default boundary $X^{STD}_{B,def}(\epsilon)$ as that value of $REV_B(\epsilon_B)$ which would generate zero dividend:

$$0 = (1-\tau)\left[X^{STD}_{B,def}(\epsilon) + n_H\left(K - \hat{X}_B(\epsilon)\right) - DEP\, e^{\hat{r}T} - F_B\right] + \tau I_{B,E}. \tag{2}$$

Simplifying, we find that the default boundary increases linearly in $\hat{X}_B(\epsilon)$:

$$X^{STD}_{B,def}(\epsilon) = DEP\, e^{\hat{r}T} + F_B - n_H(K - \hat{X}_B(\epsilon)) - \left(\frac{\tau}{1-\tau}\right) I_{B,E}. \tag{3}$$

Combining equations (1)-(2) and including an indicator function to emphasize that dividends are non-negative, dividends paid to bank shareholders are:

$$DIV^{STD}_B(\epsilon,\epsilon_B) = (1-\tau)\left(REV_B(\epsilon_B) - X^{STD}_{B,def}(\epsilon)\right)\ \mathbf{1}_{\left(REV_B(\epsilon_B) > X^{STD}_{B,def}(\epsilon)\right)}. \tag{4}$$

So far, the futures contract has been specified in terms of two exogenous parameters (K, n_H). To preclude arbitrage opportunities, these parameters must be chosen so that the date-0 present value of the futures contract is zero:

$$\begin{aligned} 0 = {} & \mathbb{E}^Q_0\left[n_H\left(\hat{X}_B(\epsilon) - K\right)\left(\mathbf{1}_{(K>\hat{X}_B(\epsilon))} + \mathbf{1}_{(K<\hat{X}_B(\epsilon))}\mathbf{1}_{\left(DIV^{STD}_B(\epsilon,\epsilon_B)>0\right)}\right)\right] \\ & + \mathbb{E}^Q_0\left[\max\left(0, (1-\alpha_B)REV_B(\epsilon_B) - F_B - DEPe^{\hat{r}T}\right)\ \mathbf{1}_{(K<\hat{X}_B(\epsilon))}\ \mathbf{1}_{\left(DIV^{STD}_B(\epsilon,\epsilon_B)=0\right)}\right]. \end{aligned} \tag{5}$$

The date-0 present value of bank equity and bank debt can therefore be expressed as

$$V_{B,E}(0) = e^{-rT}\, \mathbb{E}^Q_0\left[DIV^{STD}_B(\epsilon,\epsilon_B)\right] \tag{6}$$

$$V_{B,D}(0) = e^{-rT}\, \mathbb{E}^Q_0\left[F_B\ \mathbf{1}_{\left(DIV^{STD}_B(\epsilon,\epsilon_B)>0\right)} + Recov\ \mathbf{1}_{\left(DIV^{STD}_B(\epsilon,\epsilon_B)=0\right)}\right], \tag{7}$$

where we have defined the recovery in the event of default as:

$$\begin{aligned} Recov = {} & \mathbf{1}_{(K>\hat{X}_B(\epsilon))}\max\left(0,\ (1-\alpha_B)\left[REV_B(\epsilon_B) + n_H(K-\hat{X}_B(\epsilon))\ \right] - DEP\, e^{\hat{r}T}\right) \\ & + \mathbf{1}_{(K<\hat{X}_B(\epsilon))}\max\left[0,\ \min\left(F_B,\ (1-\alpha_B)REV_B(\epsilon_B) - DEP\, e^{\hat{r}T}\right)\right]. \end{aligned} \tag{8}$$

Here we identify the premia that banks pay for deposit insurance. If a bank defaults, the amount the FDIC will have to pay out, if positive, equals:

$$X_{INS} = DEP\, e^{\hat{r}T} - (1-\alpha_B)\left(REV_B(\epsilon_B) + n_H(K-\hat{X}_B(\epsilon))\ \mathbf{1}_{(K>\hat{X}_B(\epsilon))}\right). \tag{9}$$

Assuming banks pay fair value for this insurance, the insurance premium paid at date-0 is:

$$V_{INS}(0) \quad = \quad e^{-rT}\, \mathbb{E}_0^Q \left[X_{INS}\, \mathbf{1}_{(X_{INS}>0)}\, \mathbf{1}_{(DIV_B^{STD}(\epsilon,\epsilon_B)=0)} \right]. \tag{10}$$

The bank covers both the amount loaned to firms I_B and deposit insurance $V_{INS}(0)$ through a combination of deposits, issuing risky debt, and issuing equity:

$$DEP + I_{B,E} + I_{B,D} \quad = \quad I_B + V_{INS}(0). \tag{11}$$

Interpreting the bank as a project, shareholders choose the bank's risk management policy parameters (K, n_H) and capital structure parameter F_B in order to maximize the date-0 net present value of this project, subject to the no-arbitrage constraint specified in equation (5):

$$\mathrm{NPV}_B \quad = \quad \max_{(K,n_H,F_B)} \left(V_{B,E}(0) - I_{B,E} \right). \tag{12}$$

III Limitations of Futures Contracts for Hedging Systematic Risk.

In the optimal banking policy literature, it is common practice to specify returns as *separable* in systematic and idiosyncratic shocks. For example, returns are often specified as arithmetic Brownian motion:

$$\begin{aligned} S_{i,T} &= \sigma\, z_{M,T} + \sigma_i\, z_{idio,T} \\ M_T &= \sigma\, z_{M,T}, \end{aligned} \tag{13}$$

where $z_{M,T}$ represents an aggregate shock, $z_{idio,T}$ represents an idiosyncratic shock, $S_{i,T}$ represents unhedged returns on an individual bank, and $M_T = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} S_{i,T}$ represents unhedged returns on the financial-sector index.

Consider the date-T value of a bank that uses a futures contract to hedge bank-index risk:

$$\begin{aligned} V_T &= S_{i,T} + n_H\,(K - M_T) \\ &= n_H K + \sigma_i\, z_{idio,T} + \sigma\, z_{M,T}\,(1 - n_H), \end{aligned} \tag{14}$$

where $n_{_H}$ is the number of shares of futures contract that the bank takes a short position in, and K is the futures price, which is chosen so that the date-0 value of the futures contract is zero. We see that when returns are distributed normally, choosing a hedge ratio $n_{_H}$ equal to unity eliminates systematic risk regardless of the outcome of the idiosyncratic shock.

However, when returns are specified as *nonseparable* in aggregate and idiosyncratic shocks, as is the case when they are specified to follow *geometric* Brownian motion, then standard futures contracts are unable to fully hedge aggregate risk. Indeed, as noted by Chowdry and Schwartz (2016), under this specification, the hedge ratio that eliminates aggregate risk depends upon the outcome of the idiosyncratic shock, which is not known at the inception of the contract. This leads firms whose objective is to minimize financial distress costs to take on a relatively conservative hedge ratio.

To see this result intuitively, we generalize the model above to:

$$
\begin{aligned}
M_{_T} &= e^{\sigma\, z_{M,T} - \frac{\sigma^2}{2}T} \\
S_{_{i,T}} &= e^{\sigma\, z_{M,T} - \frac{\sigma^2}{2}T + \sigma_i\, z_{idio,T} - \frac{\sigma_i^2}{2}T} \;=\; M_{_T}\, e^{\sigma_i\, z_{idio,T} - \frac{\sigma_i^2}{2}T},
\end{aligned} \tag{15}
$$

Again, consider the date-T value of a bank that uses a standard futures contract to hedge bank-index risk:

$$
\begin{aligned}
V_{_T} &= S_{_{i,T}} + n_{_H}\left(K - M_{_T}\right) \\
&= n_{_H} K + M_{_T}\left(e^{\sigma_i\, z_{idio,T} - \frac{\sigma_i^2}{2}T} - n_{_H}\right),
\end{aligned} \tag{16}
$$

Now, *if* the firm knew at date-0 what the value of the idiosyncratic shock $z_{_{idio,T}}$ would be, then it is clear from equation (16) that the investor could eliminate market-risk by choosing a hedge ratio equal to $n_{_H} = e^{\sigma_i\, z_{idio,T} - \frac{\sigma_i^2}{2}T}$. However, the choice of the hedge ratio needs to be chosen at date-0 before the idiosyncratic shock is observed. Importantly, note that the lowest portfolio values $V_{_T}$ occur when the idiosyncratic shock term $e^{\sigma_i\, z_{idio,T} - \frac{\sigma_i^2}{2}T}$ falls below the chosen hedge ratio $n_{_H}$, and the market portfolio $M_{_T}$ performs well. Chowdry and Schwartz (2016) show that it is this effect which justifies that the bank choose a rather conservative (i.e., low) value for the hedge ratio $n_{_H}$ when minimizing financial distress costs. Note, however, that if banks use contingent-convertible (CoCo) futures to hedge financial-sector risk, the convertible feature allows them to issue equity rather than pay out losses in precisely these situations, which explains why firms can take on a more aggressive hedging policy when this bail-in feature is present. Indeed, we demonstrate in Figure 2 of the paper that the use of a CoCo futures to hedge financial-sector risk significantly reduces bank default probabilities when the industry shock is negative (i.e., when the payoff on the futures contract to the bank is positive), but does not increase bank default probabilities when the industry shock is positive (although current shareholders may be negatively impacted by a dilution effect).

Importantly, we note that while Chowdry and Schwartz (2016) identify a limitation of standard futures contracts when returns are specified as log-normal, we show in the Appendix that CoCo futures contracts are superior to standard futures contracts for reducing default risk even when returns are specified as separable in systematic and idiosyncratic shocks so that standard futures can perfectly hedge systematic risk. Intuitively this is because there exists states of nature in which the aggregate return is sufficiently high (implying banks owe something on their futures contract) and the bank-idiosyncratic shock is sufficiently low that the bail-in feature associated with CoCo futures contracts allows the bank to avoid bankruptcy by covering its loss on the futures contract via equity issuance rather than cash outflow.

IV Augmented Philippon and Wang (2021) Model

Philippon and Wang (PW, 2021) investigate a framework in which a negative externality is imposed on the economy whenever the banking system is undercapitalized. Their model assumes that at date-0, each bank $i = (1, 2, ...N)$ chooses its risk-management policy parameter $x_i \in [0, 1]$ in order to maximize its equity value. Larger values of x_i decrease date-1 capital in those "normal" states in which the banking system is well-capitalized (i.e., where the marginal benefit of an additional unit of bank capital is low), but increase capital in those "crisis" states in which the system is undercapitalized (i.e., where the marginal benefit of an additional unit of bank capital is high). Combined, these properties generate a non-linear tradeoff that leads to an inverted-U relation between the date-0 value of bank equity and its policy parameter x_i. Denote (x^*_B) as the value of (x_i) which maximizes date-0 bank equity value in the absence of bailouts. Without any incentives, banks do not internalize the negative externalities they impose on the economy, and thus would choose ($x_i = x^*_B$). In contrast, the social planner takes these externalities into account, and thus wishes to structure date-1 bailouts to incentivize banks at date-0 to choose values for the risk-management policy parameters (x_i) that are larger than (x^*_B).

In the absence of bailouts, the economy suffers welfare losses at date-1 whenever aggregate bank capital $\tilde{R}$ falls below some given threshold K. PW show that welfare can be improved if bailouts are provided in these states of nature, and that the optimal bailout size is (weakly) increasing in the gap ($K - \tilde{R}$). Moreover, they show that if these bailouts are handed out *symmetrically*, then banks may be incentivized to choose risk-management policy parameters (x_i) that are even *lower* than in the no-bailout case (x^*_B), which are already too low from a welfare standpoint. Intuitively, this is due to moral hazard, as banks internalize the fact that the size of the bailout is a decreasing function of their policy choice.

The authors then demonstrate that if the social planner structures the bailout as a tournament in which larger portions of the bailout go to banks that have higher relative performance during times of crisis, then banks can be incentivized to choose values of x_i higher than x^*_B. Moreover, they demonstrate that if costs associated with the bailout are sufficiently low, then a first-best policy can be obtained. Note, however, that the label "first-best" is in reference to a specific framework. As such, if that framework is augmented in some sense, then that same policy in the augmented framework might be only "constrained first-best" or equivalently, second-best.

Indeed, in this section we augment the PW economy so that, in addition to choosing their risk-management policy $\{x_i\}$, banks are also permitted to invest in $\{n_i\}$ shares of a futures contract written on the financial-sector index. Hence, in this augmented economy, banks have *two* policy decisions at their disposal that can transfer capital from normal states to crisis states. As we demonstrate below, not only does allowing banks to have access to futures contracts increase welfare, but in fact these contracts completely supplant the risk-management policy $\{x_i\}$, as banks can accomplish the same goal of transferring capital from normal states to crisis states more cheaply using futures. Moreover, if the model is calibrated so that an "average" performance of the banking industry ($\mathrm{E}[R] = \overline{R}$) falls above the crisis threshold K, then the introduction of a futures market creates an economy that completely eliminates the need for bailouts. In contrast, the PW economy without futures contracts suffer welfare losses either through negative externalities associated with bank undercapitalization, or through costs associated with bailouts (or both).

The intuition for these results is the following: in the absence of a bailout, the date-0 value of bank equity as a function of its policy parameter x_i takes an inverted-U shape, with a unique optimum at x^*_B. The implication of this shape is that, for values of $x_i > x^*_B$, the present value of the capital transferred out of the normal states is larger than the present value of the capital transferred into the crisis state, leading to lower equity values. Now, in the presence of a well-specified bailout, banks can be incentivized to raise their policy choice above x^*_B, as the gain from the bailout can be made larger than their loss from operations. Importantly, however, when banks are also given access to futures contracts, they can accomplish this same goal of transferring capital from normal states to crisis states *costlessly*. As such, if banks are incentivized to transfer capital to crisis states, they will do so by choosing $x_i = x^*_B$, and then by using futures contracts to accomplish this transfer. Indeed, we show that the central planner, using an arbitrarily small "bailout," can incentivize banks to hedge 100% of aggregate risk. The implication is that the banking system as a whole will have "average" capital ($R_H(\tilde{s}) = \overline{R}$) at date-1 across all aggregate states of nature ($\tilde{s}$). Because ($\overline{R} > K$), it follows that financial crises never occur in economies in which banks have access to futures contracts.

IV.1 Optimal Policies in the Absence of Bailouts

In this economy, all agents are risk-neutral, and the risk-free rate is zero. Hence, the date-0 price of any asset is equal to its expected date-1 cash flows. For each bank-i, there are two sources of risk. Aggregate risk $\tilde{s} = \{0, 1\}$ takes on one of two values, with the probability of the normal state ($\tilde{s} = 0$) equal to (p_0), and the probability of the crisis state ($\tilde{s} = 1$) equal to ($1 - p_0$). Idiosyncratic risk is distributed uniformly $\pi(\tilde{\theta}_i = \theta_i) = \frac{1}{2}\mathbf{1}_{(\theta_i \in [-1,1])}$ over the range $\theta_i \in [-1, 1]$, and thus has an expected value of zero.

We assume that all banks are ex-ante identical. In particular, at date-0, each bank-i chooses its risk-management control parameter $x_i \in [0,1]$, and is subject to the same specification for its date-1 "unhedged" capital across the two aggregate states:

$$\begin{aligned} r_U(\tilde{s}=0,\tilde{\theta}_i|x_i) &= r_0 - b_0 x_i - c_0 x_i^2 + \sigma\tilde{\theta}_i \\ r_U(\tilde{s}=1,\tilde{\theta}_i|x_i) &= r_1 + b_1 x_i + \sigma\tilde{\theta}_i, \end{aligned} \tag{17}$$

with all parameters (r_0, b_0, c_0, σ, r_1, b_1) specified as weakly positive. As such, larger values of $x_i \in [0,1]$ decrease capital in the normal state ($\tilde{s}=0$) but increase capital in the crisis state ($\tilde{s}=1$). Further, because we specify the parameter c_0 to be strictly positive, expected cash flows are concave-down in x_i in this economy.

We follow PW by focusing on symmetric solutions in which all banks ultimately choose the same policy parameter $\{x_i\} = x$. Also for simplicity, assume that the number of banks N is sufficiently large that aggregate unhedged bank capital $R_U(\tilde{s}|x) = \frac{1}{N}\sum_{i=1}^{N} r_U(\tilde{s},\tilde{\theta}_i|x)$ is independent of the idiosyncratic shocks, namely:

$$\begin{aligned} R_U(\tilde{s}=0|x) &= r_0 - b_0 x - c_0 x^2 \\ R_U(\tilde{s}=1|x) &= r_1 + b_1 x. \end{aligned} \tag{18}$$

The state-dependent payoff of one share of the futures contract is:

$$FUT(\tilde{s}) \;=\; \left(R_U(\tilde{s}|x) - \overline{R}_U(x)\right), \tag{19}$$

where the futures price is

$$\overline{R}_U(x) \;\equiv\; \mathrm{E}_0\left[R_U(\tilde{s}|x)\right] \;=\; \left[p_0 R_U(\tilde{s}=0|x) + (1-p_0) R_U(\tilde{s}=1|x)\right]. \tag{20}$$

Note that, by construction, the date-0 present value of the futures contract is zero.

In terms of its policy parameters (x_i, n_i), the state-dependent hedged capital for bank-i is:

$$\begin{aligned} r_H(\tilde{s}=0,\tilde{\theta}_i|x_i,\,n_i) &= r_0 - b_0 x_i - c_0 x_i^2 + \sigma\tilde{\theta}_i + n_i\left(R_U(\tilde{s}=0|x) - \overline{R}_U(x)\right) \\ r_H(\tilde{s}=1,\tilde{\theta}_i|x_i,\,n_i) &= r_1 + b_1 x_i + \sigma\tilde{\theta}_i + n_i\left(R_U(\tilde{s}=1|x) - \overline{R}_U(x)\right). \end{aligned} \tag{21}$$

There are two important features to note here. First. the date-0 equity value of each bank $S_0(x_i, n_i)$, which equals its expected date-1 capital, is *independent* of the choice of futures contracts n_i:

$$\begin{aligned} S_0(x_i, n_i) &= \mathrm{E}_0\left[r_H(\tilde{s}, \tilde{\theta}_i | x_i, n_i)\right] \\ &= p_0\left[r_0 - b_0 x_i - c_0 x_i^2\right] + (1-p_0)\left[r_1 + b_1 x_i\right]. \end{aligned} \tag{22}$$

Thus, in the absence of bailouts, although the bank is indifferent to its position in futures contracts $\{n_i\}$,[1] it has a unique optimal choice for its other risk-management policy:

$$x_B^* = \left(\frac{1-p_0}{p_0}\right)\frac{b_1}{2c_0} - \frac{b_0}{2c_0}, \tag{23}$$

where we have introduced the notation $x_i^* \equiv x_B^*$ in order to emphasize that all banks $i \in (1, 2, ...N)$ choose the same risk-management policy.

The second important property to note is that, if each bank-i chooses the policies $\left(x_i^* = x,\, n_i^* = -1\right)$, then it has hedged all aggregate risk and thus its date-1 capital is independent of the aggregate state-$\tilde{s}$:

$$r_H(\tilde{s}, \tilde{\theta}_i | x, n_i = -1) = \overline{R}_U(x) + \sigma\tilde{\theta}_i \qquad \forall \tilde{s}. \tag{24}$$

It is convenient to denote the hedged aggregate bank capital as $R_H(\tilde{s}|x, n) = \frac{1}{N}\sum_{i=1}^{N} r_H(\tilde{s}, \tilde{\theta}_i|x, n)$. As such, hedged aggregate bank capital is:

$$\begin{aligned} R_H(\tilde{s} = 0|x, n) &= R_U(\tilde{s} = 0|x) + n\left(R_U(\tilde{s} = 0|x) - \overline{R}_U(x)\right) \\ R_H(\tilde{s} = 1|x, n) &= R_U(\tilde{s} = 1|x) + n\left(R_U(\tilde{s} = 1|x) - \overline{R}_U(x)\right). \end{aligned} \tag{25}$$

Note that if $(n = -1)$, then aggregate bank capital equals its weighted average $\overline{R}_U(x)$ in both states of nature. Further note that this property would hold even if we specify multiple "crisis" states and multiple "normal" states. That is, if banks choose $(n = -1)$, then aggregate bank capital equals its weighted average $\overline{R}_U(x)$ across all states of nature.

[1] In the spirit of PW, we assume that positions in the futures contract are regulated to the point that hedged capital remains above the level of deposits. This imposes a lower and upper limit on permitted values for n_i.

In summary, in the absence of bailouts and without access to futures markets, all banks would choose ($x_i = x_B^*$). Moreover, in the absence of bailouts but with access to futures markets, banks still choose ($x_i = x_B^*$), and are indifferent to their choice of number of shares of futures contracts n_i. We illustrate these properties graphically in Figure 1.[2] Due to this indifference, we demonstrate below that, even with an arbitrarily small financial incentive, banks can be coaxed into hedging all systematic risk via futures contracts.

IV.2 Optimal Bailout Policies

When banks can access futures contracts to hedge financial-sector risk, the objective of the social planner can be expressed as:

$$\left(M^*_{(\tilde{s})}, x^*_G, n^*\right) \;=\; \underset{(M_{(\tilde{s})}, x, n)}{\operatorname{argmax}} \left\{ \mathrm{E}_0 \left[R_H(\tilde{s}|x,n) + \min\left(0, R_H(\tilde{s}|x,n) + M_{(\tilde{s})} - K\right) - \gamma M^2_{(\tilde{s})} \right] \right\}. \quad (26)$$

The first term is the date-0 present value of banks net of any funds received from bailouts. The second term is the negative externality that banks impose on the economy in those states of nature in which their capitalization ($R_H(\tilde{s}|x,n) + M_{(\tilde{s})}$) falls below a threshold K. The third term is the economic cost of the bailout. Increasing the size of the state-dependent bailout $M_{(\tilde{s})}$ creates a tradeoff in that it reduces the negative externality but also increases the cost of the bailout. Similarly, increasing x^*_G above x^*_B creates a tradeoff that reduces the pre-bailout present value of banks, but also reduces the negative externality. The social planner in the original PW economy faces the same objective, but is constrained to set the number of shares of the futures contract to zero (i.e., ($n^* = 0$)).

Because the first term is independent of $M_{(\tilde{s})}$, it is convenient to denote the sum of the second and third terms on the right side of equation (26) as[3]

$$\mathcal{V}(M^*_{(\tilde{s})}) \;=\; \max_{\{M_{(\tilde{s})}\}} \left[\min\left(0, (R_H(\tilde{s}|x,n) + M_{(\tilde{s})} - K)\right) - \gamma M^2_{(\tilde{s})} \right]. \quad (27)$$

[2] A key difference between the PW economy and the GS and APPR economies is that there are no tax benefits to debt in the PW economy. As such, in the absence of bailouts, bank-equity value is independent of the hedge ratio parameter n_i in the PW economy. In contrast, it is concave in n_H in the GS and APPR economies.

[3] Another tractable model specifies the bailout costs as linear: $-\gamma M_{(\tilde{s})}$. In this case, if $\gamma < 1$, then it is optimal to choose $M^*_{(\tilde{s})} = (K - R_{(\tilde{s})})$, which completely eliminates the negative externality term, and thus only the direct bailout cost term remains. In contrast, when $\gamma > 1$, then it is optimal to choose $M^*_{(\tilde{s})} = 0$.

When $(R_H(\tilde{s}|x,n) > K)$, there is no negative externality, and thus the optimal bailout is zero. Hence, in this case $\mathcal{V}(M^*_{(\tilde{s})}) = 0$, which equals its largest possible value. With that said, it is convenient in this case for the central planner to offer a "financial incentive" to banks equal to some positive constant ϵ per bank, which ultimately will be set to an arbitrarily small value. In contrast, when $(R_H(\tilde{s}|x,n) < K)$, there are two regions distinguished by bank capitalization. The first region characterizes "small crisis states" in which it is optimal to fully bail out the banking industry $\left(M^*_{(\tilde{s}=1a)} = (K - R_H(\tilde{s}|x,n))\right)$ so that all negative externalities have been eliminated, leaving the total per-bank cost to be captured by the last term $\gamma(M^*_{(\tilde{s}=1a)})^2 = \gamma(K - R_H(\tilde{s}|x,n))^2$. The second region $\left(M^*_{(\tilde{s}=1b)} = \frac{1}{2\gamma}\right)$ characterizes "extreme crisis states" in which quadratic bailout costs become too exorbitant to fully bail out the banking industry. As such, the social planner chooses the bailout to minimize the combined costs associated with negative externalities and direct bailout costs. We summarize these results in Table (I).

Table I: Optimal Bailout Policies

Capitalization Region	$M^*_{(K-R_{\tilde{s}})}$	$\mathcal{V}\left(M^*_{(K-R_{\tilde{s}})}\right)$
Normal: $(R_{\tilde{s}} - K) > 0$	ϵ	$-\gamma\epsilon^2$
Small Crisis: $0 > (R_{\tilde{s}} - K) > -\frac{1}{2\gamma}$	$(K - R_{\tilde{s}})$	$-\gamma(K - R_{\tilde{s}})^2$
Extreme Crisis: $-\frac{1}{2\gamma} > (R_{\tilde{s}} - K)$	$\frac{1}{2\gamma}$	$\frac{1}{4\gamma} - (K - R_{\tilde{s}})$

IV.3 Optimal Policy with Access to Futures Market

Given its objective as specified in equation (26), we claim that the social planner would like to mandate that all banks choose $(x^*_G = x^*_B,\ n^* = -1)$, implying that each bank would hedge 100% of its exposure to bank-index risk. Indeed, we have already shown in equations (22)-(23) that the first term, which captures pre-bailout bank equity value, is maximized when $(x^*_G = x^*_B)$, and is independent of the choice of n^*. Moreover, under our specification that $\left(\overline{R}_H(x^*_B) > K\right)$, the choice $n^* = -1$ implies that the second and third term sum to $(-\gamma\epsilon^2)$, which vanishes in the limit $\epsilon \to 0^+$.[4] Further, recall that $\mathcal{V}(M^*_{(\tilde{s})})$ has an upper bound of

[4] Even without this specification, it is straightforward to show that the optimal policy is still $(x^*_G = x^*_B,\ n^* = -1)$.

zero. Hence, the choice ($x^*_G = x^*_B$, $n^* = -1$) generates the largest possible values for *each* of the three terms in equation (26), and is thus the social-optimal policy.

Here we show that the central planner can incentivize banks to follow this social-optimal policy. The total "financial incentive" is ($NM^*_{(K-R_{\tilde{s}})}$), where N is the number of banks, and $M^*_{(K-R_{\tilde{s}})}$ is the optimal per-bank bailout. Define N^* as the number of banks that receive a share of the total financial incentive, and thus $\Pi = \left(\frac{N^*}{N}\right)$ equals the fraction of banks that receive their share of the incentive. Thus, if each bank that satisfies the criterion specified below receives $\left(\frac{M^*_{(K-R_{\tilde{s}})}}{\Pi}\right)$, then the total incentive equals $\left(\frac{M^*_{(K-R_{\tilde{s}})}}{\Pi}\right) N^*$, which equals $NM^*_{(K-R_{\tilde{s}})}$, as required.

The central planner's criterion is that, in order to receive its (equal) share of the financial incentive, the bank's date-1 capital must fall in the range:

$$r_H(\tilde{s}, \tilde{\theta}_i | x_i, n_i) \in \left[(\overline{R}_U(x^*_B) - \sigma), (\overline{R}_U(x^*_B) + \sigma) \right]. \tag{28}$$

Importantly, note that regardless of what other banks do, if bank-i chooses ($x^*_i = x^*_B$, $n^*_i = -1$), then both its stock value net of the bailout is optimized (recall equations (22)-(23)), and it is guaranteed to receive a share of the bailout. This is because the bank's idiosyncratic shock is restricted to the region $\tilde{\theta}_i \in (-1, 1)$, and thus its capital is guaranteed to satisfy the criterion to receive their fraction of the bailout. As this argument holds for all banks $i \in (1, 2, ...N)$, it follows that all banks will in fact choose this strategy, implying that in equilibrium $\Pi = 1$. More importantly, this bailout policy incentivizes all banks to hedge financial-sector risk, leading to an economy that never suffers welfare losses due to negative externalities, and thus $M^*_{(K-R_{\tilde{s}})} = \epsilon$. Moreover, since this argument holds independent of the size of the bailout ϵ, the social planner will set ϵ to an arbitrarily small positive value. Hence, the value-maximizing behavior of banks ultimately leads to a framework with no bailouts.[5]

In contrast, for the original PW economy, which is equivalent to this economy when the number of shares of futures contract (n_i) is constrained to equal zero, the optimal policy choice x^*_{PW} is larger than x^*_B. As noted previously, choosing a value $x > x^*_B$ reduces bank

[5] In this calculation, we assume that banks take the decisions of other banks as given when choosing their optimal policy choices. Hence, for example, we do not allow banks to collude in order to generate a larger bailout.

value, but reduces the present value of the negative externality even more. Still, even along the optimal policy, the sum of the second and third terms will be negative, as is always the case when there is a bailout of finite size. Hence, both the first term $\mathrm{E}_0\left[R_H(\tilde{s}|x,n)\right]$ and the combined second and third terms $\mathrm{E}_0\left[\mathcal{V}(M^*_{(\tilde{s})})\right]$ of the objective function in equation (26) are lower in this case compared to the case when banks have access to the futures market. Hence, that augmented PW economy in which banks have access to futures contracts is associated with higher welfare.

References

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Figure 1: PW Risk Management Tool vs. Futures Contracts

This figure plots relations between date-0 value of bank equity and date-1 crisis-state bank capital. Banks have two policy tools to transfer bank capital from the date-1 normal state to the date-1 crisis state, namely, the risk-management policy x_i of PW, and the number of futures contracts n_i written on the financial-sector index. It is convenient to first set n_i to zero. Moving from left to right, the black line demonstrates that when x_i increases from zero to x_B^*, both the value of bank equity and the amount of crisis capital increase. Then, "starting" at $(n_i = 0, x_i = x_B^*)$, we consider two different policies to increase crisis-state bank capital further. The red dashed line continues to set $n_i = 0$, but increases x_i from x_B^* to one. This curve shows that when banks transfer capital from the normal state to the crisis state by increasing x_i beyond x_B^*, date-0 bank equity decreases. In contrast, the blue dotted line shows that when x_i is set to x_B^* but n_i is decreased from zero to minus one, banks are able to transfer capital from the normal state to the crises state without impacting date-0 bank equity value. The implication is that if banks are incentivized (e.g., via bailouts) to increase crisis capital beyond that value characterized by the policies $(n_i = 0, x_i = x_B^*)$, they will do so by maintaining $(x_i = x_B^*)$ and reducing n_i. The intuition is that, in order to move a given amount of capital to the crisis state, the bank is required to give up less bank-capital in the normal-state by reducing n_i than by increasing x_i above x_B^*.